HYDRODYNAMICS IN CHANNELS WITH POROUS WALLS

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An experimental study has been made concerning the hydrodynamics of turbulent air flow through porous channels of uniform cross section with drain or feed along the path. Formulas are proposed for calculating the variations in pressure and flow rate along such a channel.

A growing interest has been noted in recent years concerning the hydraulic characteristics of channels with porous walls.

The variation of pressure along a channel with drain or feed is determined according to the equation of fluid flow at a variable rate along the path [1]:

$$\frac{dp}{\rho} + \beta w dw + w^2 d\beta + \frac{\beta w (w - \theta)\, dG}{G} + \lambda \frac{w^2}{2} \cdot \frac{dx}{D} = 0. \tag{1}$$

It is not possible to integrate Eq. (1), inasmuch as no reliable test data are available today pertaining to the coefficients β, λ, and the value of θ.

Several authors [2-4] assumed, without any justification, that the drain (delivery) velocity or the feed (supply) velocity of the fluid mass is perpendicular to the mainstream direction, i.e., that $\theta = 0$.

It would be more logical to assume that θ is proportional to the local fluid velocity in the channel, i.e., that $\theta = aw_x$.

Neither do researchers agree on how to calculate the flow coefficients: the momentum coefficient β and the friction drag coefficient λ. Test data for coefficient β are very scarce and contradictory [3, 5]. A few suggestions on estimating the values of coefficient λ are given in [5, 6].

An attempt to solve Eq. (1) was made in [4] for the case of fluid delivery through a perforated pipe wall, assuming $\theta = 0$ and without accounting for the variability of the local coefficient of hydraulic drag in the holes.

TABLE 1. Basic Geometrical Dimensions of Test Models

Porosity, ε_f	Diameter of holes d, mm	Number of holes per row	Spacing between holes s, mm	Length of active segment L, mm
0,00723	1,0	4	10,0	150; 250; 400; 600; 800; 1000; 1250; 1500
0,01445	1,0	4	5,0	150; 250; 400; 600; 800; 1000; 1250; 1500
0,0289	1,0	8	5,0	150; 250; 400; 600; 800; 1000; 1250; 1500
0,0578	1,0	16	5,0	150; 250; 400; 600; 800; 1000; 1250; 1500
0,1156	1,0	32	5,0	150; 250; 400; 600;
0,1156	4,0	4	10,0	150; 250; 400; 600

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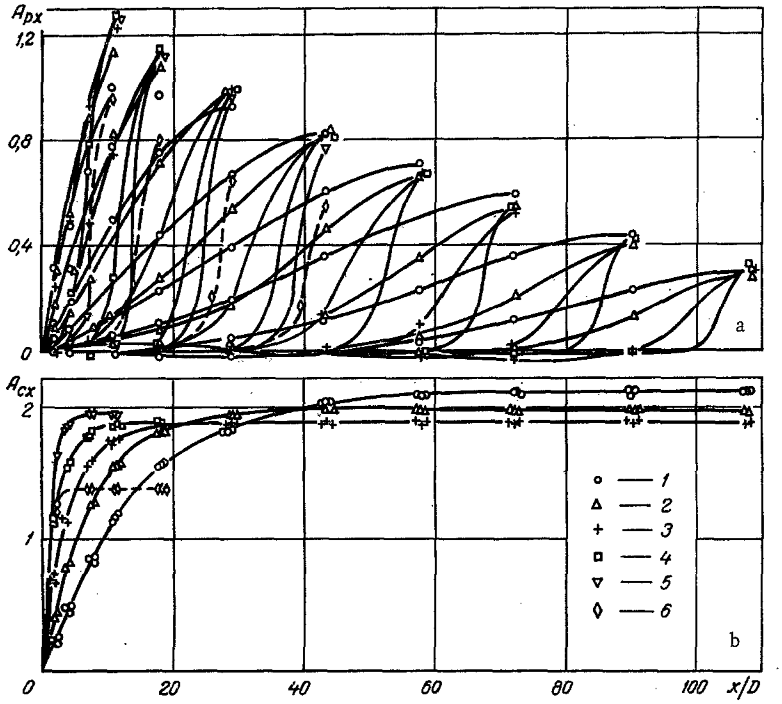

Fig. 1. A_{px} (a) and A_{cx} (b) as functions of x/D, d/D, and ε_f ($Re_0 = 5.0 \cdot 10^4$). Solid lines for d/D = 0.0722 [1) ε_f = 0.00723; 2) 0.01445; 3) 0.02892; 4) 0.05784; 5) 0.1156]. Dashed lines for d/D = 0.289 [6) ε_f = 0.1156].

Most experiments were performed with liquid or gaseous coolants supplied through a porous wall.

Most interesting, from the point of view of hydrodynamics in a porous channel, is the study made in [5]. The results of that study apply, however, to small relative lengths (L/D < 18) of an active channel with such a high drag as to ensure a uniform gas feed across the porous surface. No attention was paid at all to the hydrodynamic effects in many studies concerning the heat transfer and the friction drag with coolant injection into a turbulent boundary layer [7].

In [3] were reported results of an experimental study, and a method was proposed for calculating the hydraulic drag in a turbulent fluid stream through channels of uniform section with drain (feed) across a porous wall. The test data were generalized there on the basis of $\theta = 0$ and β = const. Although most tests were performed with a uniform feed or drain along the channel, the authors assumed that the mathematical relations would apply also to any other kind of flow rate distribution along a porous channel.

In this study the authors tried to experimentally determine the mode of pressure and flow rate variation along a porous channel of uniform section carrying turbulent air with drain or feed along the path, at various porosity levels and at various lengths of the active channel segment. The test object was a circular brass pipe with an inside diameter D = 13.8 mm, whose porosity was defined by the number of holes through the wall (d = 1 mm and d = 4 mm). At a total pipe length of 1800 mm, the longest active segment was 1500 mm and the flow stabilization segment was over 20 diameters long.

The geometrical parameters of the test models of porous pipes are given in Table 1. The length of the active segment was varied within the range L/D ≈ 10.8–108 by means of a movable plunger pushed in from the dead-end side.

A differential manometer or a micromanometer installed at sampling points read the entrance pressure into the active segment and the pressure variation along the porous pipe, which were then recorded accordingly.

Each model was tested under four different operating modes, namely with four values of the Reynold's number at the entrance section to the active segment, within the range $Re_0 = 3 \cdot 10^4 – 2 \cdot 10^5$ ($w_0 \approx$ 35–240 m/sec). Tests were performed with both air drain and air feed. The respective results are shown in Fig. 1a, b for one flow mode ($Re_0 = 5.0 \cdot 10^4$). The Euler number has been plotted here along the axis of ordinates: Eu = A_{px} (drain) and Eu = A_{cx} (feed).

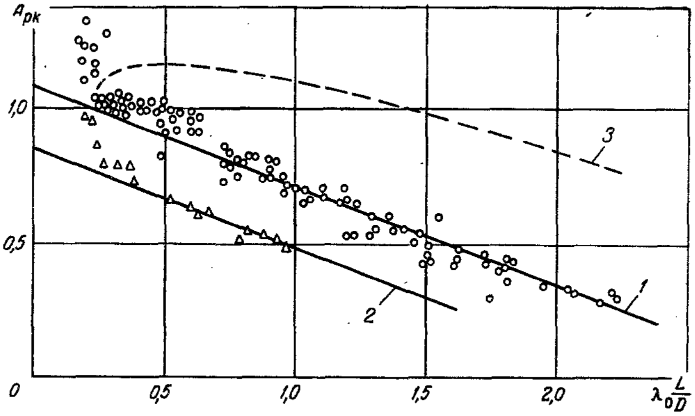

Fig. 2. A_{pk} as a function of $\lambda_0 L/D$: 1) $d/D = 0.072$; 2) $d/D = 0.29$; 3) according to the formulas in [3].

According to the derived relations, the total pressure drop along a porous segment with drain $A_{pk} = \Delta p_k/(\rho w_0^2/2)$ does not depend on the channel porosity $\bar{\varepsilon_j}$ (or the parameter $\bar{f}$), but is determined by the relative channel length L/D, by the relative diameter of holes d/D, and by the flow mode (Reynolds number Re_0). For given values of L/D and d/D, at the same time, the mode of pressure variation along a channel is largely affected by the porosity level.

The total relative pressure drop (A_{pk}) based on all drain tests is shown in Fig. 2 as a function of the total hydraulic drag coefficient $\lambda_0 L/D$. It is evident here that the test data fit closely enough into the equation

$$A_{pk} = \frac{\Delta P_{pk}}{\frac{\rho w_0^2}{2}} = b - \frac{\lambda_0}{2.75} \cdot \frac{L}{D}, \tag{2}$$

which follows from Eq. (3)

$$A_{px} = \frac{\Delta p}{\frac{\rho w_0^2}{2}} = X(2-X)b - \frac{\lambda_0 L}{2.75D}\left[1-(1-X)^{2.75}\right]. \tag{3}$$

describing the pressure variation during a drain flow [8]. Equation (3), in turn, follows from the fundamental equation (1) with $\theta = aw_x$, $\beta = \text{const}$, $w_x = w_0(1-X)$, $b = a\beta$, and $\lambda_0 = 0.3164/\sqrt[4]{Re_0}$.

The values of coefficient $b = f(d/D)$ had been obtained experimentally in [8] for the case of a flow with unilateral drain.

It is evident from Fig. 2 that the test points for short channels (L/D < 18) lie farther and farther above the calculated straight line. This can probably be explained by the effect of a dead end on the flow pattern in short channels. On the same diagram is shown a curve (dashed line) calculated by the method in [3] from test values for a uniform drain.

While measurements and calculations agree fairly well in the case of short channels, such as those tested by the authors (L/D = 12), the error of calculations by the method in [3] for any other than uniform drain distribution along the channel increase rapidly with increasing L/D ratio.

An evaluation of test results has yielded the following empirical formula for the pressure variation with drain across a porous wall:

$$A_{px} = \frac{\Delta p}{\frac{\rho w_0^2}{2}} = aX^m, \tag{4}$$

where

$$a = b - \frac{\lambda_0}{2.75} \cdot \frac{L}{D},$$

and

$$m = 1.25\bar{f} - 2.5 \quad \text{for} \quad \bar{f} \geqslant 3.0;$$

$$m = 0.3\bar{f} + 0.4 \quad \text{for} \quad \bar{f} \leqslant 3.0. \tag{5}$$

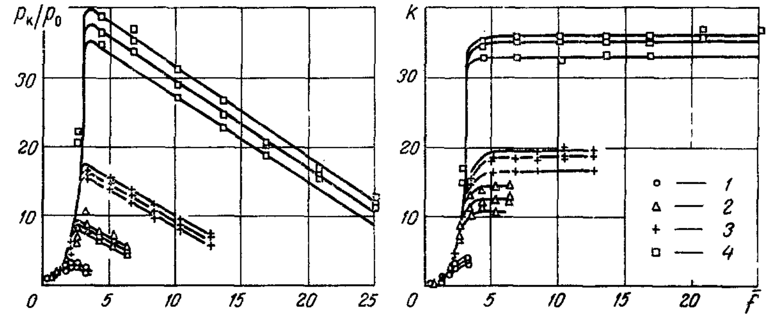

Fig. 3. Curves of k and p_k/p_0 vs f (for drain mode): 1) ε_f = 0.00723; 2) 0.01445; 3) 0.02892; 4) 0.05784.

The value of coefficient b should be taken as 1.0-1.1 for d/D < 0.1-0.12.

When $\bar{f} < 3.0$ (especially $\bar{f} < 1$), formula (3) gives a better agreement with test data.

In this study the total pressure drop along the channel was of the same order of magnitude as the hydraulic drag through the porous wall and the distribution of fluid flow rate (velocity) through holes at various channel sections was not uniform.

The degree of hydraulic nonuniformity can be expressed in terms of the parameter $\eta_h = v/\bar{v}$, where $\bar{v} = G_0/\rho \Sigma f_h$. On the other hand,

$$\eta_G = \sqrt{\frac{p_0 + \Delta p}{p_0 + \bar{\Delta p}}} \quad \text{or} \quad \eta_G = \sqrt{\frac{1 + kA}{1 + k\bar{A}}}. \tag{6}$$

Accepting that

$$A = \frac{\Delta p}{\frac{\rho w_0^2}{2}} = aX^m,$$

we will find the mean relative pressure drop along a porous collecting pipe

$$\bar{A} = \frac{\Delta p}{\rho \frac{w_0^2}{2}} \int_0^1 aX^m dX = \frac{a}{m+1}. \tag{7}$$

Then Eq. (6) becomes

$$\eta_G = \frac{v}{\bar{v}} = \sqrt{\frac{1 + kaX^m}{1 + k\frac{a}{m+1}}}, \tag{8}$$

where $k = (\rho w_0^2/2)/p_0$.

Curves of k vs $\bar{f}$ are shown in Fig. 3 for various porosity levels and various flow modes in a delivery channel. When $\bar{f} > 3$, evidently k becomes independent of $\bar{f}$, depending only on the porosity and on the flow mode.

For a channel with a porosity $0.12 > \varepsilon_f \geq 0.012$ and $\bar{f} > 3.0$, parameter k can be determined from the equation

$$k = \exp\left(\frac{0.04}{\lambda_0} + 25\varepsilon_f\right). \tag{9}$$

When $k \leq 0.1-0.2$, the air delivery is almost uniform throughout the length of a porous channel.

In Fig. 3 is also shown the relation $p_k/p_0 = \varphi(\bar{f})$. Evidently the pressure recovery is maximum when $\bar{f} \approx 3.0$.

The test data obtained for a porous channel with suction (intake channel) are shown in Fig. 1b and Fig. 4. According to Fig. 1b, increasing the relative hole size (d/D) will result in a smaller pressure drop along the porous channel segment. The relative pressure drop along a porous channel with injection can be determined according to the expression in [8], on the basis of Eq. (1),

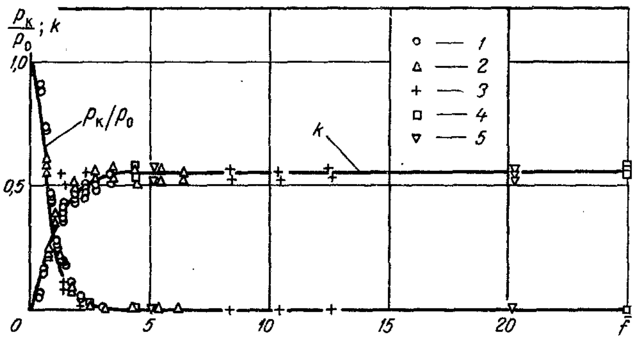

Fig. 4. Curves of $\rho w_0^2/2p_0 = \varphi(\bar{f})$ and $p_k/p_0 = \psi(\bar{f})$ (for intake mode): 1) $\varepsilon_f = 0.00723$; 2) 0.01445; 3) 0.02892; 4) 0.05784; 5) 0.1156.

$$A_{cx} = \frac{\Delta p}{\rho \frac{w_0^2}{2}} = 2cX(2-X) + \frac{\lambda_0 L}{2.75D}[1-(1-X)^{2.75}]. \tag{10}$$

Equation (10) is based on the following assumptions: β = const; $w_x = w_0(1-X)$; $\theta = a_1 w_x$. Coefficient $c = (2-a_1)/2 \cdot \beta$ accounts for the effect of added fluid mass on the mainstream structure. The values of this coefficient can be determined from test data in [8]. When $d/D < 0.25$, then $c \cong 1.0$. The total relative pressure drop along a porous channel segment is

$$A_{ch} = \frac{\Delta p_k}{\rho \frac{w_0^2}{2}} = 2c + \frac{\lambda_0 L}{2.75D}. \tag{11}$$

It follows from the curves in Fig. 4 that, unlike in delivery channels, intake channels are characterized by parameters k and p_k/p_0 almost independent of the porosity but determined by the parameter $\bar{f}$ and the flow mode only.

The channel parameters begin to be independent of $\bar{f}$ already from $\bar{f} = 2.5$ on. Within the range of independence $p_k/p_0 = 0$ and $k = 0.5$. When $\bar{f} > 2.5$, the velocity through holes is $v = 0$. Since $\bar{f} = 4L/D \cdot \varepsilon_f$ for channels of circular cross section, hence $\bar{f} > 2.5$ and thus $x/D \geq 0.625/\varepsilon_f$ such a channel becomes almost idle. With increasing porosity, the active channel segment will decrease in length.

The pressure variation along an intake channel can be calculated with the aid of Eq. (10). This equation is valid, however, only for porous channels with the ratio $L/D < 0.625/\varepsilon_f$.

NOTATION

p is the static pressure;
$p_x = p(x)$;
ρ is the density;
w is the mean-over-the-section velocity in the channel;
$w_x = w(x)$;
v is the radial velocity through holes in the channel wall;
G is the mass flow rate;
L is the total length of the porous channel;
D is the equivalent diameter of the channel;
d is the diameter of holes through the porous wall;
F_{ch} is the area of flow section of the channel;
Σf_h is the area of holes through the channel wall;
$\bar{f} = \Sigma f_h/F_{ch}$;
ε_f is the porosity of the channel;
s is the spacing of holes in the channel wall;
λ is the friction drag coefficient;
Re is the Reynolds number
x is the coordinate;
$X = x/L$ $(0 \leq x \leq L,\ 0 \leq x \leq 1)$;

X is the relative coordinate along the channel;
$p_0 = p(0)$;
$w_0 = w(0)$;
$\lambda_0 = \lambda(0)$;
$Re_0 = Re(0)$.

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